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A PARAMETRIC MODEL FOR MULTISPECTRAL SCANNERS

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I. ABSTRACT

Efficient acquisition and utilization of remotely sensed data requires an extensive a priori evaluation of the performance of the basic data collection unit, the multispectral scanner. The objective is the development of a fully parametric technique to theoretically evaluate the systems response in any desired operational environment and provide the necessary information in selecting a set of optimum parameters.

In this paper the multispectral scanner spatial characteristics are represented by a linear shift-invariant multiple-port system where the N spectral bands comprise the input processes. The scanner characteristic function, the relationship governing the transformation of the input spatial and hence spectral correlation matrices through the systems, is developed. Specific cases for Gaussian point spread functions are examined.

The integration of the scanner spatial model and a parameter classification error estimator provides the necessary technique to evaluate the performance of a multispectral scanner. A set of test statistics are specified and the corresponding output quantities computed by the characteristic function. Two sets of classification accuracies, one at the input and one at the output are estimated. The scanner's instantaneous field of view is changed and the variation of the output classification performance monitored.

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II. INTRODUCTION

An important class of remote sensing systems has as its primary goal the collection in selected spectral bands of reflected or emitted electromagnetic energy. This data is then used to identify and characterize the sources of the radiation. A widely used earth resources data gathering system of this type is the electro-optical scanning radiometer commonly referred to as a multispectral scanner (MSS). The signal degradations caused by various transformations within the scanner subsystem strongly affect system performance. The finite scanner aperture and the atmospheric and quantization noise are but some of the contributing factors. The optimization of the entire set of interactive parameters within the scanner can be quite involved. The classification accuracy obtained by processing the actual data is necessarily suboptimum due to the aforementioned degradation sources. A reference probability of misclassification (PMC) could be defined by analyzing the performance using the reflected signal at the scanner input, even though this signal is obviously inaccessible. By simulating a theoretical model for the MSS the classification error rate can be evaluated and compared at the scanner input and output thereby establishing an upper bound on the system performance in the context of the defined index of performance. Arbitrary spatial resolution can be specified and its interactive relationship with the SNR and PMC studied.

The projected algorithm will have several capabilities. The most important one is the ease of parameter manipulation. Variation of the scanner spatial resolution will cause the output statistics to be modified with a corresponding variation in the estimate of the classification error. Similarly, variations in the population separability at the scanner input and the resulting interaction with the PMC can be studied.

This built-in flexibility is a desirable and almost imperative feature of the scanner system modeling. A specific example is the class statistics manipulation. The generation of a new data set, with prescribed statistics, requires appropriate software and, depending on the data base magnitude, can be potentially time consuming. The alternative in the proposed algorithm is to supply the data statistics instead of the data samples.

Modeling of the MSS by a linear system opens the way to the application of existing techniques in system theory. Since the classification accuracy of MSS data is totally a function of class statistics under the Bayes rule, examination of the random process transformation carried out by the scanner PSF can provide much useful information. Topics of particular interest are

1. Effect of the scanner IFOV on population statistics.
2. Effect of data spatial correlation on the classification accuracy.
3. Effect of signal-to-noise ratio on classification accuracy.
4. Trade off between spatial resolution and SNR.
5. Effect of spatial resolution on classification accuracy.
6. The interactive relationship between IFOV, spatial correlation, class statistics, SNR and classification accuracy.

III. MSS SPATIAL MODEL

The averaging operation performed by the scanner point spread function can be modeled by a linear shift-invariant multiple-input, multiple-output system. Input signals consist of N random processes in N spectral bands corrupted by atmospheric noise and scattering. Each input is linearly transformed by the scanner PSF and additional detector and pre-amp noise further contribute to the signal degradation.

Fig. 1 is a block diagram of this spatial model. h(x,y) is the two dimensional PSF to be specified for any desired system. In particular where the MSS is concerned, the assumption of a Gaussian shaped IFOV has been widespread. The justification for this is essentially satisfactory experimental results and perhaps equally important is the mathematical convenience of this model. Note that the results obtained hereafter are fundamentally independent of the functional form of the PSF. However, using this assumption, it is frequently possible to obtain closed form expressions and to make comparisons with alternate methods a majority of which adhere to the same assumption.

In a two dimensional plane a Gaussian PSF is specified by the following relationship

$$h(x,y) = c_1 e^{-\frac{x^2}{r_o^2}} e^{-\frac{y^2}{r_o^2}} \qquad (1)$$

The important parameter is r_o, the PSF's characteristic length, which in effect determines the ultimate ground resolution and noise content of the collected data. Increasing r_o results in a deterioration of the resolution but improvement in the SNR. An important property of h(x,y) is its separability in the cross and along-track directions resulting in some simplifications of the analytical relationships governing the scanner operation. In practice, h(x,y) is truncated at some point, (e.g., 0.1 h(0,0)) to keep the computation time down. The normalizing constant c_1, provides a unity gain for this averaging operation.

A. MSS STATISTICAL MODEL AND SPATIAL CORRELATION

As the input random processes undergo a linear transformation, so do their statistical properties. In order to investigate the various interactive relationships outlined previously, an understanding and knowledge of the signal flow through the scanner is essential.

Relating the statistics of the multispectral signal at the scanner output to the corresponding part at the input can be accomplished in various ways. It has been pointed out that a two dimensional convolution is equivalent to a matrix multiplication in which one matrix is block circulant[1]. Let F and G be the input and output matrices arranged in P^2x1 column vectors. Then they are related by

$$\underline{G} = \underline{HF} \qquad (2)$$

where PSF matrix $\underline{H}$, has the following structure

$$
\underline{H} = \begin{bmatrix} H_o & H_{P-1} & \cdots & H_1 \\ H_1 & H_o & \cdots & H_2 \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ H_{P-1} & H_{P-2} & & H_o \end{bmatrix}
$$

Each element in $\underline{H}$ is itself a P x P matrix. For a particular case, a selected number of fields can be chosen and processed by (2) to produce the G matrix followed by the calculation of a pooled auto- and cross-spectral correlation matrix.

This method has the advantage of requiring no a priori spatial information yet its data dependent nature makes the results of any study limited to the particular data set used. The more general approach, providing possibly closed form expressions for the quantities desired, is the application of linear system theory techniques to the MSS. This, however, requires some a priori specification of data properties in an algebraic form, the main item being the spatial correlation model.

Comparatively speaking, spectral classification has been much more widespread than spatial classification, resulting in less than full attention to the spatial properties of remotely sensed data. It has been suggested, however, that the experimentally observed correlation functions approximately follow a decaying exponential[2,3]. This assumption implies a Markov model for the spatial characteristics of the data. Let $\underline{R}_k$ be the spatial correlation matrix of the kth spectral band

$$
\underline{R}_k = [r_{ij}] \quad i,j = 0, 1, \ldots, n_o-1 \tag{3}
$$

Under the two assumptions: (a) Markov correlation structure; and (b) separability along the cross-track and along-track directions, $\underline{R}_k$ can be specified as follows

$$
\underline{R}_k = [r_{ij}] = \rho_{x_k}^{i} \, \rho_{y_k}^{j} \qquad i,j = 0, 1, \ldots, n_o-1 \tag{4}
$$

where ρ_{x_k} and ρ_{y_k} are the adjacent pixel correlation coefficients along the respective directions given by

$$
\rho_{x_k} = e^{-a_{kk}}
$$

$$
\rho_{y_k} = e^{-b_{kk}} \tag{5}
$$

Similarly, the spatial crosscorrelation matrix between two bands p and q is defined as

$$
\underline{R}_{pq} = [r_{ij}] = \rho_{x_{pq}}^{i} \, \rho_{y_{pq}}^{j} \qquad i,j = 0, 1, \ldots, n_o-1 \tag{6}
$$

where

$$
\rho_{x_{pq}} = e^{-a_{pq}} \qquad \rho_{y_{pq}} = e^{-b_{pq}} \tag{7}
$$

With the correlation model defined, the output spectral covariance matrix can be specified. Let $\underline{R}_{g_i g_j}$ and $\underline{\Sigma}_g$ be the output spatial correlation matrix between spectral bands i and j and output covariance matrix, respectively, then

$$
\underline{\Sigma}_g(i,j) = [\underline{R}_{g_i g_j}(0,0)] \qquad i,j = 1, 2, \ldots, N \tag{8}
$$

Note that when considered over the ensemble of all the bands, matrix $\underline{R}_g$ is an (n_o x N) (n_o x N) partitioned matrix, given by

$$
\underline{R}_g = \begin{bmatrix} [\underline{R}_{g_1 g_1}] & [\underline{R}_{g_1 g_2}] & \cdots & [\underline{R}_{g_1 g_n}] \\ [\underline{R}_{g_2 g_1}] & [\underline{R}_{g_2 g_2}] & \cdots & [\underline{R}_{g_2 g_N}] \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ [\underline{R}_{g_N g_1}] & \cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot & & [\underline{R}_{g_N g_N}] \end{bmatrix} \tag{10}
$$

where $[\underline{R}_{ij}]$ is the n_o x n_o spatial correlation matrix. Σ_g however, is only a function of zero lag elements of $\underline{R}_g$, $\underline{R}_{g_i g_j}(0,0)$. Therefore, only N x N out of (n_o x N) (n_o x N) entries of $\underline{R}_g$ need be calculated. It is clear that the spectral correlation matrix is a small subset of spatial correlation matrices whose elements have the following locations.

$$
\underline{\Sigma}_g(i,j) = \underline{R}_g((i-1)n_o, (j-1)n_o) \qquad i,j = 1, 2, \ldots, N \tag{10}
$$

IV. SCANNER CHARACTERISTIC FUNCTION

In order to determine the effects of different scanner IFOV's and their interaction with the classification accuracy of a data set, it is essential that the required output covariance matrices be parametrically represented in terms of known input quantities. In the above it was noted that the entire spectral covariance matrix is specified if the appropriate spatial correlation functions are known. Let f(x,y), g(x,y) and h(x,y) denote the input and output random processes associated with any two matching bands and the scanner PSF, respectively. It is well known that the above quantities are related by a convolution integral.

$$g(x,y) = \iint f(x-\lambda_1, y-\lambda_2)\, h(\lambda_1,\lambda_2)\, d\lambda_1 d\lambda_2 \tag{11}$$

In order to derive specific results, a spherically symmetric Gaussian PSF is considered. The spatial correlation matrix describing the scene is a two sided exponential.

A. GAUSSIAN SCANNER PSF

The PSF and spatial correlation model are given by

$$R_{ff}(\tau,\eta) = \rho_o^{|\tau|} \rho_o^{|\eta|}$$

$$h(x,y) = c_1\, e^{-\frac{x^2}{r_o^2}}\, e^{-\frac{y^2}{r_o^2}} \tag{12}$$

where $\rho_o = e^{-a}$ is the adjacent pixel correlation assumed equal along the horizontal and vertical directions. This assumption is not in contradiction with the fact that in a Landsat data set sample-to-sample correlation is higher than line-to-line correlation because of the closer physical distance between the samples. In the continuous domain, such as this formulation, where theoretically equally spaced lines and columns can exist, there is little reason for assuming different pixel-to-pixel correlation along each direction. Two quantities, c_1 and r_o specify the PSF where c_1 is a normalizing constant providing unity gain and r_o is the filter's characteristic length, closely related to the IFOV.

With the parameters of the problem defined, the scanner output correlation function can be expressed as;

$$S_{gg}(u,v) = S_{ff}(u,v)\,|H(u,v)|^2 \tag{13}$$

where S(u,v) is spectral density. Let $M(u,v) = |H(u,v)|^2$, then

$$R_{gg}(\tau,\eta) = R_{ff}(\tau,\eta) * m(\tau,\eta) \tag{14}$$

$$m(\tau,\eta) = \frac{\pi c_1^2 r_o^2}{2}\, e^{-\frac{\tau^2}{2r_o^2}}\, e^{-\frac{\eta^2}{2r_o^2}} \tag{15}$$

Using the separability property of the functions involved and carrying out the integration,

$$R_g(\tau,\eta) = \left[e^{\frac{a^2 r_o^2}{2} - a\tau}\, Q\left(a r_o - \frac{\tau}{r_o}\right) + e^{\frac{a^2 r_o^2}{2} - a\tau}\, Q\left(a r_o - \frac{\tau}{r_o}\right) \right]^2 \tag{17}$$

The above relationship can be easily modified to cover the case of unequal pixel-to-pixel correlation along cross-track and down-track directions. If $R_{ff}(\tau,\eta)$ is given by

$$R_{ff}(\tau,\eta) = e^{-a|\tau|}\, e^{-b|\eta|}$$

Then it follows that

$$R_{gg}(\tau,\eta) = \left[e^{\frac{a^2 r_o^2}{2} - a\tau}\, Q\left(a r_o - \frac{\tau}{r_o}\right) + e^{\frac{a^2 r_o^2}{2} + a\tau}\, Q\left(a r_o + \frac{\tau}{r_o}\right) \right] \times \left[e^{\frac{b^2 r_o^2}{2} - b\eta}\, Q\left(b r_o - \frac{\eta}{r_o}\right) + e^{\frac{b^2 r_o^2}{2} + b\eta}\, Q\left(b r_o + \frac{\eta}{r_o}\right) \right] \tag{18}$$

Note that since the input process f(x,y) has a unity variance $R_{gg}(0,0)$ is in effect a weighting by which any input variance will be multiplied to produce the corresponding output spectral variance. The right hand side of (18), therefore, can be considered as a weighting function associated with any multiband scanner to relate input and output statistics. Denote this function by $W_s(\tau,\eta,a,b)$, the scanner characteristic function.

The next item of interest is the output crosscorrelation among channels. This quantity, designated by $R_{g_i g_j}(\tau,\eta)$, is a straight forward extension of the method just described. Again assuming a Markov or exponential structure governing the crosscorrelation function between channels

$$R_{f_i f_j}(\tau,\eta) = s_{f_i f_j}\sigma_{f_i}\sigma_{f_j} e^{-a_{ij}|\tau|} e^{-b_{ij}|\eta|} \tag{19}$$

and following identical techniques, the crosscorrelation coefficient between channels i and j at the scanner output is given by

$$s_{g_i g_j} = \frac{W_s(0,0,a_{ij},b_{ij})}{W_s^{\frac{1}{2}}(0,0,a_{ii},b_{ii})\, W_s^{\frac{1}{2}}(0,0,a_{jj},b_{jj})} s_{f_i f_j} \tag{20}$$

where $s_{f_i f_j}$ is the input crosscorrelation coefficient. Therefore, the band-to-band correlation coefficients are identical at scanner input and output provided spatial auto- and crosscorrelation functions at the input are equivalent, i.e., $a_{ii} = a_{ij}$, $b_{ii} = b_{ij}$.

Evaluating $W_s(\tau,\eta,a,b)$ for all values of τ and η can complete the entire output spatial matrix $\underline{R}_g$. The Bayes classifier, however, is not a spatial classifier but, rather, is a spectral one and, as a result, the knowledge of a N x N spectral covariance matrix is sufficient for classification purposes. Using a parametric model provides a considerable flexibility in system analysis. For example, W_s can selectively supply any entry of the output spatial matrix desired. Here, $W_s(\tau,\eta,a,b)|_{\tau=\eta=0}$ can complete the output spectral covariance matrix

$$W_s(0,0,a,b) = 4e^{\frac{(a^2+b^2)}{2}r_o^2} Q(ar_o)Q(br_o) \tag{21}$$

When the input random process is a two spectral band data set, the output spectral correlation matrix, $\underline{S}_g$ is given in terms of $\underline{S}_f$ as follows:

$$\underline{S}_f = \begin{bmatrix} 1 & s_{f_1 f_2} \\ & 1 \end{bmatrix}$$

$$\underline{S}_g = \begin{bmatrix} 1 & \dfrac{W_s(0,0,a_{12},b_{12})}{W_s^{\frac{1}{2}}(0,0,a_{11},b_{11})W_s^{\frac{1}{2}}(0,0,a_{22},b_{22})} s_{f_1 f_2} \\ & 1 \end{bmatrix} \tag{22}$$

It is clear that, depending on the particular value of W_s, the output correlation matrices, and hence, classification accuracies will be modified. The variations of W_s as a function of scene correlation and scanner spatial parameters can be very illuminating. For a Gaussian scanner PSF, W_s is plotted vs. the sample-to-sample correlation for a fixed line-to-line correlation. The IFOV is used as a running parameter, Fig. 2. The adjacent sample correlation coefficient ranges from a near white noise 0.1 to total correlation of 1 (constant signal amplitude). The selected line-to-line correlation is 0.8.

Examination of the variations of W_s reveals several important features. Since $0 \le W_s \le 1$, the output channel variances are always smaller than the corresponding input quantity. This is a widely observed feature of any scanner system due to the averaging property of the system's PSF. Fig. 1 shows that for any combination of scene correlation W_s is a decreasing function of IFOV size. Also, for a fixed IFOV, W_s is an increasing function of scene correlation. The spatial properties of a scene play a significant role in the overall system performance which is not readily obvious. One of the well known properties of linear systems with random inputs is the reduction of the output variance/input variance ratio (W_s) as the PSF is widened. Specifically, with everything else fixed, a process having a moderate scene correlation will undergo a tighter clustering around its mean than an otherwise identical process with highly correlated spatial characteristics. On the extreme side of the correlation scale with small pixel-to-pixel correlation, the ratio of the output to input variance is very negligible.

V. CLASSIFICATION ACCURACIES AT THE MSS OUTPUT

A hypothetical three population three feature data set is used for test purposes. The set is completely specified by the following spectral correlation matrices corresponding to two visible and one infrared band.

$$\underline{S}_{f_1} = \begin{bmatrix} 1 & 0.75 & 0.15 \\ & 1 & 0.45 \\ & & 1 \end{bmatrix},$$

$$\underline{S}_{f_2} = \begin{bmatrix} 1 & 0.8 & 0 \\ & 1 & 0.1 \\ & & 1 \end{bmatrix},$$

$$\underline{S}_{f_3} = \begin{bmatrix} 1 & 0.94 & 0.15 \\ & 1 & 0.05 \\ & & 1 \end{bmatrix}$$

These statistics were selected after examinations of the correlation matrices obtained for different cover types[4]. An attempt was made to choose correlation structures that would approximately represent some typical cases, albeit crudely. Whether this is true or not, however, has little bearing on the results of this simulation process. The data is processed through the scanner for two different adjacent sample correlations of 0.5, and 0.95. For each case, the IFOV is varied from 1 to 8 high resolution pixels. The output spectral statistics are computed using the scanner characteristic function followed by the estimation of Bayes classification accuracies using the ACAP algorithm. The results are shown in Fig. 3 and 4.

The variations of the output probabilities of correct classification are in complete agreement with those projected by the characteristic function. The most notable feature is the inverse relationship between the scene spatial correlation and the slope of $\hat{P}_{c|\omega_i}$ vs. IFOV at the output. When the scene is spatially highly uncorrelated such as Fig. 3, $\hat{P}_c$ gained 16.2% by increasing the IFOV from 1 to 2 pixels wide, whereas, the same increase in IFOV produced a gain of only 0.9% when $\rho_x = 0.95$. This behavior can be predicted from the variations of W_s vs. ρ_x. Referring to Fig. 2 where W_s is plotted, it is observed that the one step reduction in input variance gets progressively smaller toward higher scene correlations. For the test case under study where any reduction of the class variances along a feature axis can contribute to increased separability, the aforementioned property of W_s accounts for the changing slope of $P_{c|\omega_i}$ over the ensemble of the scene spatial correlations.

VI. SUMMARY AND CONCLUSIONS

The objective of this study was to employ the ACAP error estimation technique and MSS model in an integrated parametric package that would produce the theoretical response of the MSS in a fully controllable environment. The results presented are not intended to be exhaustive but rather to demonstrate the method and to illustrate general trends in the system response. It is constructive to compare the patterns observed with those obtained by other simulation techniques.

A parallel study aimed at the same objectives is reported by Landgrebe[5]. High resolution aircraft MSS data was considered with a cascade of simulated scanner PSF's to produce data sets with 30 m, 40 m, 50 m and 60 m ground resolutions and the classification performance was estimated for each case. The results provided less than conclusive evidence on the monotonic relationship between classification performance and the IFOV due to the very small rise in $\hat{P}_c$ as IFOV was enlarged. This conclusion can be fully understood from the theoretical curves of $\hat{P}_c$ vs. IFOV. The significant parameter, data spatial correlation, is what determines how strongly classification performance and IFOV are interrelated. As for a real data set, its spatial correlation structure is a fixed parameter. In case of high resolution aircraft data, pixel-to-pixel correlation can be as high as 0.9 or 0.95. Fig. 4 with $\rho_x = 0.95$ clearly illustrates that $\hat{P}_c$ and IFOV are indeed weakly coupled. Had the data under investigation by Landgrebe[5] been less spatially correlated, this coupling would manifest itself more strongly. For satellite data having a ρ_y of about 0.75-0.8, $\hat{P}_c$ shows considerably stronger sensitivity to variations of IFOV.

VII. REFERENCES

1. Hunt, B. R., "Deconvolution of Linear Systems by Constrained Regression and Its Reactionship to the Wiener Theory," IEEE Trans. Autom. Control, 1972.

2. Ready, P. J., and Wintz, P. A., "Multiapectral Data Compression Through Transform Coding and Block Quantization," Purdue Univ. TR-EE 72, 1972.

3. Kettig, R. L., "Computer Classification of Remotely Sensed Multispectral Image Data by Extraction and Classification of Homogeneous Objects," Purdue Univ., LARS Information Note 050975.

4. Landgrebe, D. A., et. al., Final Report, Laboratory for Applications of Remote

Sensing NASA Contract NAS9-14016, June 1, 1975 - May 31, 1976.

5. Landgrebe, D. A., et. al., "An Empirical Study of Scanner System Parameters," Purdue Univ./Laboratory for Applications of Remote Sensing, Information Note 110976, 1976.

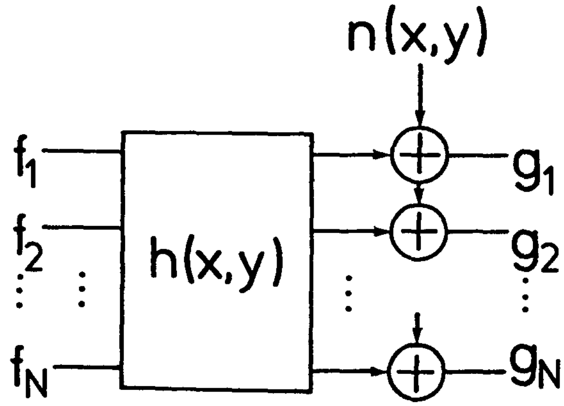

Fig. 1. MSS Spatial Model as a Linear System.

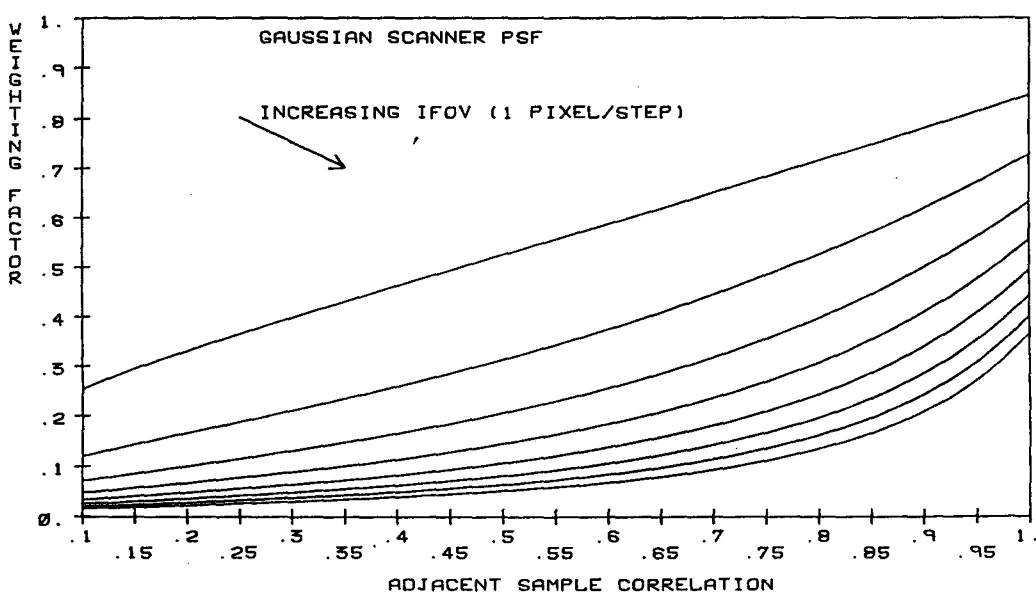

FIG. 2. SCANNER CHARACTERISTIC FUNCTION VS. SCENE CORRELATION ADJACENT LINE CORRELATION= .8

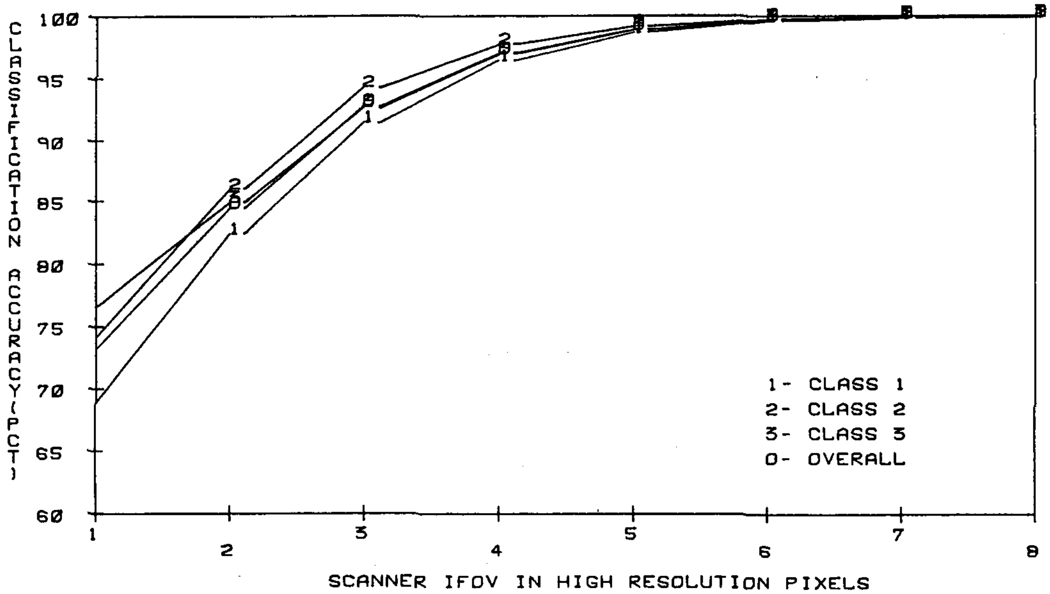

FIG. 3. SCANNER OUTPUT CLASSIFICATION ACCURACY VS. IFOV
ADJACENT SAMPLE CORRELATION= .5

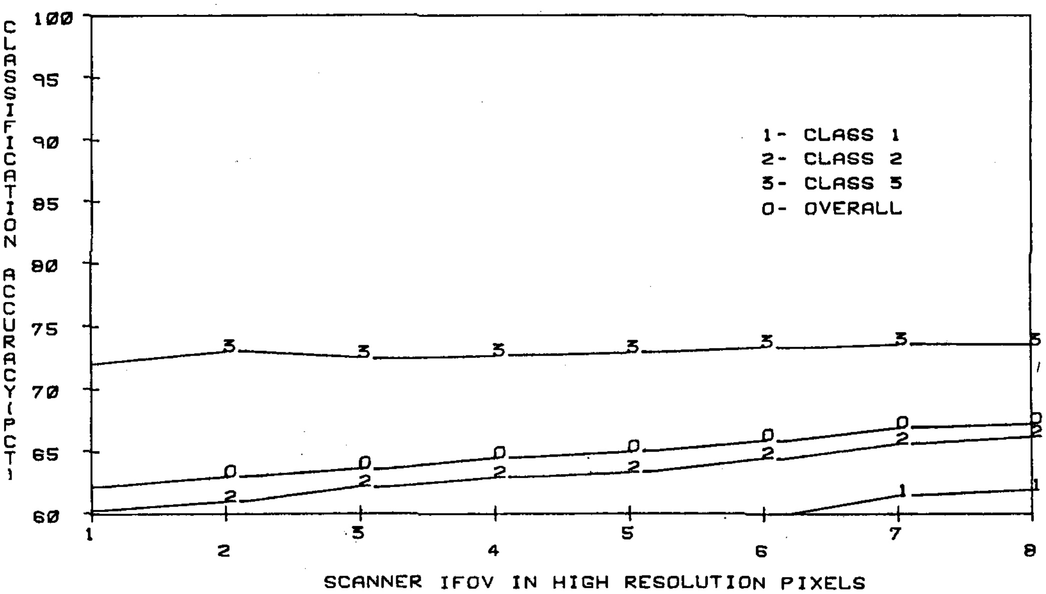

FIG. 4. SCANNER OUTPUT CLASSIFICATION ACCURACY VS. IFOV
ADJACENT SAMPLE CORRELATION= .95